IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

Some Tests Procedures for Scale Differences

Pronita Gogoi[1], Bipin Gogoi[2]

Research Scholar, Department of Statistics, Dibrugarh University, Assam, India[1]

Professor, Department of Statistics, Dibrugarh University, Assam, India[2]

Abstract: In this paper we have considered the eleven existing tests for testing the homogeneity of variances in two-sample. The test statistics are – Cucconi, Zhang, Neyman-Pearson, O'Brien, Link, Newman, Bliss-Cochran-Tukey, Cadwell-Leslie-Brown, Overall-Woodward Z-variance, modified Overall-Woodward Z-variance and Chen W tests. Among these tests Chen W test is based on sample spacing. The comparative analysis of power of the tests is carried out for the distributions viz. normal, logistic, Cauchy and double exponential. Results are calculated by using Monte Carlo simulation technique.

Keywords- Cucconi test, Zhang test, Neyman-Pearson test, O'Brien test, Link test, Newman test, Bliss-Cochran-Tukey test, Cadwell-Leslie-Brown test, Overall-Woodward Z-variance test, modified Overall-Woodward Z-variance test and Chen W tests, simulated level and power.

1. INTRODUCTION

Equality of variances is a characteristic of the null hypothesis when homogeneous subjects are randomly assigned to the treatment and placebo (Brownie et al.,1990)and then the presence of difference in variability may indicate that the treatment produces an effect. In such situations, a test for jointly detecting location and scale changes is more appropriate than a test for the Beherens-Fisher problem. The most familiar nonparametric test for the two sample location-scale problem is the test of Lepage(1971) which is a combination of the Wilcoxon test for detecting location changes and the Ansari-Bradley test for detecting scale changes. Podgor and Gastwirth(1994) proposed a class of nonparametric tests for the location-scale problem and also extended the results of O'Brien (1988).This approach can recast as a quadratic combination of a rank test for location and a rank test for scale which generates Lepage type test. Marozzi (2009) compared several Podgor and Gastwirth tests and found the one based on the Wilcoxon test and the Mood squared rank test to be generally preferable. Neuhauser (2000) and Murakami (2007) proposed a modification of the Lepage test and concluded that their modified tests should be preferred to the classical one. Cucconi (1968) proposed a rank test that is earlier than the Lepage one. This rank test is of interest because contrary to the other location-scale tests it is not a combination of a test for location and a test for scale. It is based on squared ranks and squared contrary-ranks. Even if this test is not much familiar, a detailed simulation study by Marozzi (2009) showed that its size is very close to α and is more powerful than the Lepage test. Zhang(2006) proposed a new approach for constructing nonparametric tests for the general two-sample problem which generates as particular cases traditional tests like the Kolmogorov-Smirnov, Cramer-Von Mises, and Anderson-Darling tests, as well as new powerful tests based on the likelihood ratio. Zhang recommended the test that is the analog of the Cramer-Von Mises test because it has the simplest form within the proposed class. Marozzi (2013) considered the several nonparametric tests due to Podgor and Gastwirth, Neuhauser, Murakami, Cucconi and Zhang for the jointly detection of location and scale changes and found that Neuhauser test and Murakami test performed well.

In this paper we considered eleven test statistics viz. Cucconi test, Zhang ZH test, Neyman-Pearson test, O'Brien test, Link test, Newman test, Bliss-cochran-Tukey test, Cadewell-Leslie-Brown test, Overall-Woodword Z-variance test, Modified Overall-Woodword Z-variance test and W test. Among these tests the W test based on sample spacings. We want to study the performance of these tests under distribution viz. normal, Cauchy, logistic and double exponential. Monte Carlo Simulation techniques are used to study the performance of these test statistics.

2. TEST PROCEDURES:

Let m and n be the sample sizes of two random samples $X_1, \dots X_m$ and $Y_1, \dots, Y_n$, $N = m + n$. We assume that the elements within each sample are independently and identically distributed, and we assume independence between the two samples. Let F_1 and F_2 be the continuous distribution functions underlying the two samples. The null hypothesis is

H_0: $F_1\left(\frac{t-\mu}{\sigma_1}\right) = F_2\left(\frac{t-\mu}{\sigma_2}\right)$ for all t ϵ (-∞,∞).

We wish to test H_0, against the scale alternative

$$H_1 : F_1\left(\frac{t-\mu}{\sigma_1}\right) \neq F_2\left(\frac{t-\mu}{\sigma_2}\right)$$

2.1. *Cucconi C Test*

The Cucconi (1968) test statistic is defined as

$$C=\frac{U^2+V^2-2\rho UV}{2(1-\rho^2)} \qquad (1)$$

Where,

$$U=\frac{6\sum_{j=1}^{n} S_j^2-n(N+1)(2N+1)}{\sqrt{mn(N+1)(2N+1)(8N+11)/5}}$$

$$V=\frac{6\sum_{j=1}^{n}(N+1-S_j)^2-n(N+1)(2N+1)}{\sqrt{mn(N+1)(2N+1)(8N+11)/5}} \quad \text{and}$$

$$\rho=\frac{2(N^2-4)}{(2N+1)(8N+11)}-1$$

and S_j is the rank of Y_j in the pooled sample. Under H_0, $E_0(U)=E_0(V)=0$ and $VAR_0(U) = VAR_0(V)=1$.
Under H_0, (U,V) is centered on (0,0), whereas it is not under H_1, and the large values of C speak against H_0.Marozzi (2009) computed a table of critical values that was missing in the literature. It should be noted that without ties it makes no difference whether U and V are computed based on the data of the first or the second sample, whereas in the presence of ties, one obtains very slightly different values for C.

2.2. *Zhang ZH Test*:

The Zhang (2006) *ZH* test statistic is defined as

$$ZH=\frac{1}{N}\left(\sum_{i=1}^{m}\ln\left(\frac{m}{i-0.5}-1\right)\ln\left(\frac{N}{G_i-0.5}-1\right)+\sum_{j=1}^{n}\ln\left(\frac{n}{j-0.5}-1\right)\ln\left(\frac{N}{H_j-0.5}-1\right)\right) \qquad (2)$$

Where G_i, $i = 1, 2, \ldots, m$ and H_j, $j = 1, 2, \ldots, n$ are the ranks increasing order of each X_i and Y_j in the pooled sample. It is important to note that the *ZH* test is location, scale and shape sensitive. It is as powerful as the traditional Kolmogorov-Smirnov, Cramer-Von Mises, and Anderson-Darling tests for detecting changes in location, and more powerful for detecting changes in scale or shape. It is important to note that small values of *ZH* speak against H_0

2.3. *Neyman-Pearson Test(NP):*

This test is a ratio between arithmetic mean of all estimate of variance s_i^2 and geometric mean.

$$NP=\frac{1}{m}\sum_{i=1}^{m} s_i^2/\left(\prod_{i=1}^{m} s_i^2\right)^{\frac{1}{m}} \qquad (3)$$

where m is the number of samples,
$s_i^2=\frac{1}{n_i-1}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^2$ is the estimate of sample variance. $\bar{x}_i=\frac{1}{n_i}\sum_{j=1}^{n_i}x_{ij}$ is the mean of i-th sample, x_{ij} – j-th observation in the i-th sample. The test is right-sided. The hypothesis H_0 is rejected when $h>h_{1-\alpha}$.

2.4. *O'Brien Test (V):*

This test was introduced by O'Brien (1978) and he claimed that his test is a general method that does fairly well for behavioral science data. O'Brien (1981) states that the test is robust to data that departs from normality and it can be easily used in different ANOVA designs with equal or unequal sample sizes.
The test statistic is:

$$V=\frac{\frac{1}{m-1}\sum_{i=1}^{m} n_i(\bar{V}_i-\bar{\bar{V}}_i)^2}{\frac{1}{N-m}\sum_{i=1}^{m}\sum_{j=1}^{n_i}(V_{ij}-\bar{V}_i)^2} \qquad (4)$$

Where $\bar{V}_i=\frac{1}{n_i}\sum_{j=1}^{n_i}V_{ij}$, $\bar{\bar{V}}_i=\frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i}V_{ij}$, $N=\sum_{i=1}^{m}n_i$ and $V_{ij}=\frac{(n_i-1.5)n_i(x_{ij}-\bar{x}_i)^2}{(n_i-1)(n_i-2)}$

The test is right sided. If the test statistic exceeds the critical value the null hypothesis H_0 is rejected. When the null hypothesis is true the statistic of the O'Brien test has approximately $F_{m-1,N-m}$ distribution.

2.5. *Link Test (F^*):*

Link (1950) proposed a test is analogue of Fisher test and in this test two independent samples were drawn from normal population with the same variance. This test is therefore comparable to F test.
The test statistic is defined as

$$F^*=\frac{\omega_m}{\omega_n} \qquad (5)$$

Where $\omega_m=x_{1,max}-x_{1,min}$, $\omega_n=x_{2,max}-x_{2,min}$ is ranges of samples.

 DOI 10.17148/IARJSET.2017.411122

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

ISO 3297:2007 Certified

Vol. 4, Issue 11, November 2017

The test is two-sided. The hypothesis is rejected if $F^* > F^*_{1-\alpha/2}$ or $F^* < F^*_{\alpha/2,}$ where α is significance level, $F^*_{1-\alpha/2}$ and $F^*_{\alpha/2}$ is upper and lower critical values of statistic. The distribution of test statistic depends essentially on sample sizes.

2.6. *Newman Test (N):*

The test statistic of Newman (1939) is defined as

$$N = \frac{\omega_m}{s_n}, \qquad (6)$$

Where $\omega_m = x_{1,max} - x_{1,min}$,

$$s_n = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{2i} - \overline{x_2})^2}$$

The Newman test is two-sided. If the null hypothesis is true, the distribution of test statistics depends on sample sizes and the law of distribution of samples.

2.7. *Bliss-Cochran-Tukey Test (BCT):*

This test was introduced by Bliss, Cochran and Tukey (1956) and as analogue of the Cochran test

$$BCT = \frac{\max_{i\le j\le m}\omega_i}{\sum_{i=1}^{m}\omega_i}, \qquad (7)$$

where m is number of samples, $\omega_i = \max_{1\le j\le n_i} x_{ij} - \min_{1\le j\le n_i} x_{ij}$ is range of i^{th} sample.

The test is two-sided. The distribution of test statistic highly depends on sample sizes. As the Cochran test, the distributions of the test statistic are sensitive to departure from normality.

2.8. *Cadwell-Leslie-Brown Test (K):*

This test proposed by Leslie and Brown (1966) and is analogue to Hartley test:

$$K = \frac{\max_{1\le i\le m}\omega_i}{\min_{1\le i\le m}\omega_i}, \qquad (8)$$

where m is sample number, ω_i is range of i^{th} sample. The test is right-sided. The distribution of the Cadwell-Leslie-Brown test, as the distribution of Bliss-Cochran test statistic, depends essentially on sample sizes and law of distribution.

2.9. *Overall-Woodward Z-variance Test (Z):*

This test statistic was proposed by Overall and Woodward (1974) and they found that it performed well for normally distributed data, also produced too much Type I error in skewed distributions. The test statistics is

$$Z = \frac{1}{m-1}\sum_{i=1}^{m} Z_i^2, \qquad (9)$$

Where m is number of samples,

$$Z_i = \sqrt{\frac{c_i(n_i-1)s_i^2}{MSE}} - \sqrt{c_i(n_i-1) - \frac{c_i}{2}},$$

$c_i = 2+1/n_i$,

$MSE = \frac{1}{N-m}\sum_{i=1}^{m}\sum_{j=1}^{n_i}(x_{ij} - \overline{x}_i)^2$, n_i is size of i^{th} sample, s_i^2 is the unbiased estimate of sample variances, $N = \sum_{i=1}^{m} n_i$

If the null hypothesis H_0 is true and the samples obey normal law of distribution, the distribution of test statistic has approximately $F_{m-1,\infty}$ distribution and does not depend on sample sizes.

2.10. *Modified Overall-Woodward Z-variance Test (MZ):*

Overall and Woodward (1976) proposed modification of Z-variance test to construct test that would remain stable when sample data deviate from normality. The new values c_i depends on the sample size, skewness and the mean of kurtosis indices. They determined c to be a scaling coefficient that affects the variability of the Zi values.

The new formula for c is:

$$c_i = 2.0\left[\frac{1}{K_i}\left(2.9 + \frac{0.2}{n_i}\right)\right]^{\frac{1.6(n_i - 1.8K_i + 14.7)}{n_i}}, \qquad (10)$$

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

where n_i = The sample size of the ith subgroup, $K_i = \frac{1}{n_i-2}\sum_{j=1}^{n_i} G_{ij}^4$ -estimate of kurtosis index of i^{th} sample, $G_{ij} = (x_{ij} - \bar{x}_i)/\sqrt{\frac{n_i-1}{n_i} s_i^2}$, $\bar{K}$ - the mean of kurtosis indices.

The distribution of test statistic converges slowly to $F_{m-1,\infty}$ distribution with increased sample sizes. Even when the sample sizes are large values, the distribution of test statistic differs from $F_{m-1,\infty}$ distribution.

2.11. *Chen W – Test (W) :*

Chen (2004) proposed a test statistics W based on sample spacing's. The proposed test statistics is :

$W = a^2 + Nb^2 + c^2, N = m + n$ (11)

Where a be the number of y_j's less than $X_{(1)}$, b be the number of y_j's between $X_{(1)}$ and $X_{(n)}$ and c be the number of y_j's greater than $X_{(m)}$. $X_{(i)}$, *i = 1,2, ... , m,* be the order statistics of *X* sample. This new test statistics *W* can also be expressed in terms of $S_1, S_2, \ldots, S_{m+1}$,

$$W = S_1^2 + N(S_2 + S_3 + \ldots + S_m)^2 + S_{m+1}^2 \quad (12)$$

Where S_i = the number of y_j's in the interval $[X_{(i-1)}, X_{(i)}]$, *i =1, 2, ..., m+1,* $X_{(0)} = 0$ and $X_{(m+1)} = 1$, S_i may be called "spacing frequencies".

3. SIMULATION

To stucy the significance level and power of the test, the considered sample sizes are (*10,10*), (*10,15*), (*15,15*), (*15,20*), (*20,20*), (*20,25*), (*25,30*) and (*30,30*). We here generate random sample from normal, logistic, Cauchy and double exponential distributions. The null hypothesis of equal variance is studied along with the alternatives: (*1, 1.5*), (*1, 2*), (*1, 2.5*) and (*1, 3*). Here ***Table.1(a)*** to ***Table.1(d)*** gives the estimate of significance level of the considered tests for normal, logistic, Cauchy and double exponential distributions respectively. ***Table.2(a)*** to ***Table.2(d)*** gives the estimate of power of tests for normal, logistic, Cauchy and double exponential distribution respectively.

Table.1(a)
Significance Level of the Tests under Normal Distribution

(n_1,n_2)	C	ZH	K	N	Z	BCT	NP	V	F^*	MZ	W
(10,10)	0.097 0.042 0.003	0.101 0.065 0.009	0.100 0.050 0.010	0.100 0.050 0.010	0.099 0.045 0.006	0.100 0.050 0.010	0.100 0.050 0.010	0.104 0.046 0.008	0.100 0.050 0.010	0.100 0.050 0.010	0.121 0.057 0.012.
(10,15)	0.099 0.046 0.004	0.099 0.049 0.009	0.100 0.050 0.010	0.100 0.050 0.010	0.104 0.051 0.007	0.100 0.049 0.010	0.100 0.050 0.010	0.090 0.038 0.005	0.100 0.050 0.010	0.100 0.050 0.010	0.105 0.059 0.012
(15,15)	0.099 0.046 0.007	0.099 0.050 0.010	0.100 0.050 0.010	0.100 0.050 0.010	0.103 0.048 0.007	0.100 0.050 0.010	0.100 0.050 0.010	0.090 0.034 0.004	0.100 0.050 0.010	0.100 0.050 0.010	0.097 0.054 0.013
(15,20)	0.099 0.049 0.007	0.099 0.050 0.010	0.100 0.050 0.010	0.100 0.050 0.010	0.105 0.053 0.008	0.100 0.050 0.010	0.100 0.050 0.010	0.096 0.041 0.005	0.100 0.050 0.010	0.100 0.050 0.010	0.122 0.055 0.010
(20,20)	0.095 0.045 0.007	0.099 0.050 0.009	0.100 0.050 0.009	0.100 0.050 0.010	0.103 0.051 0.009	0.100 0.050 0.010	0.100 0.050 0.010	0.097 0.044 0.006	0.100 0.050 0.010	0.100 0.050 0.010	0.107 0.061 0.012
(20,25)	0.098 0.048 0.008	0.099 0.050 0.010	0.100 0.050 0.010	0.100 0.050 0.010	0.104 0.049 0.009	0.099 0.050 0.011	0.100 0.050 0.010	0.095 0.043 0.005	0.100 0.050 0.010	0.100 0.050 0.010	0.108 0.055 0.010
(25,30)	0.097 0.047 0.007	0.099 0.049 0.009	0.099 0.050 0.010	0.100 0.050 0.010	0.098 0.051 0.009	0.099 0.050 0.010	0.100 0.049 0.010	0.094 0.045 0.006	0.100 0.050 0.010	0.100 0.050 0.010	0.104 0.050 0.010
(30,30)	0.100 0.048 0.008	0.100 0.050 0.009	0.100 0.050 0.010	0.100 0.050 0.010	0.099 0.051 0.008	.100 .050 .010	0.100 0.049 0.010	0.095 0.045 0.006	0.100 0.050 0.010	0.100 0.050 0.010	0.101 0.053 0.011

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

Table.1(b)
Significance Level of the tests under Logistic Distribution

(n_1,n_2)	C	ZH	K	N	Z	BCT	NP	V	F^*	MZ	W
(10,10)	0.093	0.097	0.175	0.120	0.162	0.175	0.163	0.102	0.146	0.114	0.122
	0.040	0.062	0.108	0.067	0.087	0.108	0.094	0.043	0.087	0.063	0.056
	0.002	0.008	0.030	0.015	0.018	0.030	0.026	0.007	0.029	0.015	0.013
(10,15)	0.098	0.099	0.170	0.128	0.163	0.170	0.155	0.100	0.155	0.112	0.102
	0.045	0.047	0.104	0.079	0.092	0.104	0.091	0.045	0.097	0.061	0.057
	0.005	0.011	0.033	0.024	0.025	0.033	0.029	0.008	0.030	0.014	0.011
(15,15)	0.099	0.102	0.180	0.140	0.171	0.180	0.167	0.102	0.150	0.115	0.102
	0.047	0.051	0.114	0.083	0.099	0.113	0.103	0.044	0.092	0.065	0.055
	0.005	0.009	0.038	0.025	0.025	0.038	0.031	0.006	0.034	0.017	0.013
(15,20)	0.101	0.100	0.185	0.146	0.171	0.185	0.167	0.101	0.164	0.113	0.120
	0.047	0.047	0.111	0.090	0.098	0.111	0.094	0.045	0.099	0.059	0.055
	0.007	0.007	0.037	0.032	0.027	0.037	0.027	0.007	0.029	0.015	0.010
(20,20)	0.101	0.099	0.191	0.162	0.179	0.191	0.173	0.103	0.156	0.118	0.107
	0.048	0.053	0.123	0.099	0.104	0.123	0.103	0.049	0.100	0.063	0.061
	0.006	0.011	0.041	0.036	0.028	0.041	0.029	0.007	0.034	0.015	0.013
(20,25)	0.101	0.104	0.196	0.169	0.177	0.196	0.173	0.102	0.160	0.115	0.108
	0.045	0.049	0.124	0.110	0.104	0.124	0.105	0.047	0.102	0.062	0.054
	0.009	0.009	0.045	0.040	0.029	0.047	0.027	0.006	0.037	0.014	0.012
(25,30)	0.099	0.097	0.204	0.189	0.183	0.204	0.186	0.101	0.167	0.119	0.103
	0.046	0.048	0.136	0.124	0.108	0.136	0.104	0.046	0.110	0.065	0.052
	0.008	0.010	0.049	0.047	0.029	0.049	0.031	0.005	0.038	0.012	0.011
(30,30)	0.098	0.100	0.206	0.208	0.184	0.206	0.185	0.100	0.156	0.122	0.097
	0.047	0.049	0.128	0.140	0.113	0.128	0.110	0.047	0.098	0.059	0.051
	0.008	0.008	0.050	0.058	0.033	0.050	0.036	0.007	0.034	0.017	0.011

Table.1(c)
Significance Level of the Tests under Cauchy Distribution

(n_1,n_2)	C	ZH	K	N	Z	BCT	NP	V	F^*	MZ	W
(10,10)	0.093	0.090	0.666	0.386	0.681	0.666	0.681	0.057	0.363	0.315	0.120
	0.038	0.055	0.610	0.351	0.613	0.610	0.622	0.017	0.326	0.194	0.055
	0.003	0.009	0.489	0.286	0.478	0.489	0.507	0.001	0.265	0.084	0.011
(10,15)	0.095	0.102	0.670	0.348	0.700	0.670	0.696	0.060	0.440	0.382	0.104
	0.041	0.046	0.616	0.316	0.644	0.616	0.644	0.025	0.407	0.252	0.062
	0.004	0.009	0.509	0.259	0.527	0.509	0.546	0.003	0.341	0.095	0.010
(15,15)	0.102	0.100	0.709	0.436	0.732	0.709	0.731	0.044	0.377	0.467	0.098
	0.046	0.050	0.657	0.405	0.677	0.657	0.680	0.010	0.347	0.365	0.051
	0.006	0.009	0.560	0.344	0.573	0.560	0.585	0.001	0.289	0.167	0.011
(15,20)	0.096	0.096	0.721	0.396	0.747	0.721	0.745	0.046	0.449	0.501	0.117
	0.041	0.043	0.669	0.369	0.697	0.670	0.692	0.015	0.416	0.400	0.054
	0.007	0.006	0.572	0.321	0.600	0.572	0.605	0.001	0.357	0.235	0.011
(20,20)	0.099	0.099	0.747	0.454	0.770	0.747	0.769	0.041	0.408	0.564	0.103
	0.047	0.053	0.704	0.426	0.728	0.704	0.727	0.010	0.381	0.476	0.058
	0.006	0.009	0.608	0.376	0.641	0.608	0.642	0.001	0.327	0.315	0.010
(20,25)	0.101	0.104	0.751	0.403	0.780	0.751	0.778	0.039	0.460	0.585	0.103
	0.048	0.051	0.707	0.406	0.738	0.707	0.739	0.011	0.430	0.503	0.054
	0.007	0.009	0.625	0.356	0.654	0.627	0.653	0.001	0.381	0.359	0.011
(25,30)	0.098	0.102	0.773	0.471	0.802	0.773	0.803	0.036	0.447	0.636	0.100
	0.049	0.047	0.731	0.447	0.762	0.731	0.760	0.008	0.420	0.566	0.051
	0.008	0.011	0.649	0.403	0.685	0.649	0.691	0.000	0.369	0.430	0.012
(30,30)	0.104	0.103	0.775	0.507	0.809	0.775	0.810	0.035	0.413	0.661	0.091
	0.051	0.052	0.732	0.484	0.773	0.732	0.772	0.007	0.389	0.589	0.049
	0.009	0.010	0.655	0.443	0.701	0.655	0.707	0.001	0.344	0.473	0.012

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

ISO 3297:2007 Certified

Vol. 4, Issue 11, November 2017

Table.1(d)
Significance Level of the Tests under Double Exponential Distribution

(n_1,n_2)	C	ZH	K	N	Z	BCT	NP	V	F*	MZ	W
(10,10)	0.093 0.040 0.002	0.102 0.062 0.008	0.253 0.177 0.067	0.179 0.120 0.044	0.244 0.152 0.049	0.253 0.177 0.067	0.245 0.163 0.066	0.097 0.041 0.006	0.184 0.127 0.057	0.123 0.073 0.019	0.122 0.056 0.013
(10,15)	0.098 0.045 0.005	0.094 0.047 0.008	0.249 0.172 0.072	0.187 0.125 0.053	0.250 0.164 0.054	0.249 0.172 0.072	0.376 0.258 0.093	0.100 0.041 0.007	0.211 0.149 0.063	0.122 0.066 0.017	0.102 0.057 0.010
(15,15)	0.099 0.047 0.005	0.099 0.049 0.008	0.264 0.187 0.081	0.212 0.147 0.060	0.266 0.172 0.064	0.264 0.187 0.080	0.434 0.313 0.127	0.099 0.039 0.005	0.192 0.135 0.060	0.123 0.070 0.021	0.102 0.055 0.013
(15,20)	0.101 0.047 0.007	0.096 0.047 0.008	0.265 0.181 0.077	0.218 0.153 0.073	0.265 0.176 0.067	0.265 0.182 0.077	0.454 0.318 0.134	0.100 0.042 0.006	0.218 0.149 0.062	0.120 0.063 0.019	0.120 0.055 0.010
(20,20)	0.101 0.048 0.006	0.099 0.053 0.010	0.271 0.193 0.084	0.245 0.175 0.083	0.272 0.184 0.073	0.271 0.193 0.084	0.488 0.358 0.167	0.101 0.045 0.006	0.201 0.139 0.060	0.124 0.070 0.019	0.107 0.061 0.013
(20,25)	0.101 0.045 0.009	0.106 0.051 0.009	0.273 0.197 0.092	0.253 0.188 0.092	0.273 0.186 0.075	0.273 0.197 0.094	0.499 0.385 0.182	0.100 0.044 0.004	0.210 0.148 0.069	0.119 0.065 0.017	0.108 0.054 0.012
(25,30)	0.099 0.046 0.008	0.096 0.046 0.011	0.282 0.204 0.094	0.292 0.214 0.108	0.280 0.194 0.079	0.282 0.204 0.094	0.549 0.418 0.230	0.102 0.043 0.004	0.215 0.152 0.068	0.126 0.071 0.014	0.103 0.052 0.011
(30,30)	0.098 0.047 0.008	0.097 0.048 0.011	0.290 0.198 0.090	0.317 0.240 0.132	0.277 0.198 0.081	0.290 0.198 0.090	0.574 0.450 0.265	0.100 0.043 0.006	0.195 0.138 0.059	0.126 0.067 0.020	0.097 0.051 0.011

Table.2(a)
Empirical power of tests under Normal Distribution

(n_1,n_2)	(σ_1,σ_2)	C	ZH	K	N	Z	BCT	NP	V	F*	MZ	W
(10,10)	(1,1.5) (1,2) (1,2.5) (1,3)	.104 .248 .403 .541	.081 .133 .202 .281	.195 .468 .696 .841	.300 .609 .815 .919	.234 .497 .708 .843	.194 .468 .696 .840	.215 .520 .755 .887	.159 .344 .496 .604	.292 .595 .796 .905	.165 .374 .561 .689	.250 .492 .685 .807
(10,15)	(1,1.5) (1,2) (1,2.5) (1,3)	.103 .263 .448 .618	.048 .072 .119 .186	.278 .597 .817 .925	.354 .690 .873 .952	.251 .538 .754 .886	.278 .597 .817 .925	.257 .601 .838 .944	.104 .257 .406 .514	.300 .624 .836 .935	.164 .383 .578 .708	.276 .551 .755 .874
(15,15)	(1,1.5) (1,2) (1,2.5) (1,3)	.164 .417 .646 .803	.089 .213 .384 .546	.254 .598 .834 .940	.386 .756 .923 .981	.339 .677 .874 .954	.254 .598 .833 .940	.309 .717 .914 .977	.211 .505 .715 .821	.357 .708 .899 .967	.262 .602 .819 .921	.328 .652 .853 .942
(15,20)	(1,1.5) (1,2) (1,2.5) (1,3)	.166 .461 .713 .854	.072 .186 .373 .555	.329 .703 .903 .974	.437 .810 .950 .986	.367 .722 .907 .971	.329 .703 .903 .974	.345 .768 .947 .990	.202 .523 .729 .832	.369 .740 .922 .980	.257 .623 .841 .935	.348 .692 .885 .961
(20,20)	(1,1.5) (1,2) (1,2.5) (1,3)	.225 .577 .812 .921	.143 .403 .670 .835	.308 .699 .908 .975	.455 .839 .969 .994	.425 .800 .943 .986	.308 .700 .908 .975	.397 .836 .970 .995	.318 .707 .887 .946	.428 .805 .948 .988	.353 .763 .934 .983	.439 .795 .936 .984
(20,25)	(1,1.5) (1,2) (1,2.5) (1,3)	.234 .614 .854 .953	.124 .386 .667 .852	.396 .784 .948 .990	.501 .877 .980 .998	.445 .820 .959 .993	.396 .784 .948 .990	.454 .883 .985 .997	.314 .730 .900 .955	.446 .823 .961 .993	.373 .796 .951 .989	.434 .800 .950 .989

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

(25,30)	(1,1.5)	.302	.171	.436	.565	.515	.436	.536	.435	.498	.479	.487
	(1,2)	.736	.551	.840	.926	.889	.840	.940	.858	.875	.904	.867
	(1,2.5)	.930	.837	.971	.992	.985	.971	.995	.965	.981	.986	.977
	(1,3)	.982	.952	.997	.999	.998	.997	.999	.987	.998	.998	.996
(30,30)	(1,1.5)	.345	.239	.396	.585	.558	.396	.575	.505	.519	.536	.513
	(1,2)	.787	.685	.827	.940	.914	.827	.957	.912	.894	.933	.892
	(1,2.5)	.956	.923	.968	.994	.989	.968	.998	.986	.986	.994	.982
	(1,3)	.992	.984	.995	.999	.999	.995	.999	.996	.998	.999	.998

Table.2(b)
Empirical power of tests under Logistic Distribution

(n_1,n_2)	(σ_1,σ_2)	C	ZH	K	N	Z	BCT	NP	V	F^*	MZ	W
(10,10)	(1,1.5)	.091	.073	.238	.296	.234	.238	.247	.120	.314	.153	.215
	(1,2)	.205	.118	.466	.564	.497	.466	.515	.260	.571	.317	.421
	(1,2.5)	.344	.174	.662	.759	.708	.662	.723	.388	.751	.465	.600
	(1,3)	.480	.243	.796	.867	.843	.796	.851	.485	.860	.579	.729
(10,15)	(1,1.5)	.092	.047	.330	.355	.251	.330	.300	.095	.350	.151	.241
	(1,2)	.227	.067	.593	.639	.538	.593	.594	.226	.618	.318	.470
	(1,2.5)	.395	.107	.789	.822	.754	.788	.803	.351	.806	.473	.666
	(1,3)	.550	.160	.893	.912	.886	.893	.914	.445	.903	.584	.796
(15,15)	(1,1.5)	.147	.087	.303	.402	.339	.303	.344	.184	.385	.224	.279
	(1,2)	.363	.184	.587	.707	.677	.587	.682	.435	.678	.491	.560
	(1,2.5)	.579	.324	.785	.883	.874	.785	.877	.619	.850	.700	.754
	(1,3)	.736	.472	.898	.954	.954	.898	.955	.732	.933	.816	.873
(15,20)	(1,1.5)	.153	.068	.386	.442	.367	.386	.386	.169	.418	.226	.293
	(1,2)	.401	.160	.688	.763	.722	.689	.741	.411	.717	.505	.588
	(1,2.5)	.643	.305	.865	.916	.907	.865	.917	.608	.883	.707	.791
	(1,3)	.804	.471	.944	.972	.971	.944	.975	.725	.953	.826	.903
(20,20)	(1,1.5)	.194	.123	.361	.478	.425	.631	.422	.249	.455	.290	.358
	(1,2)	.504	.325	.674	.805	.800	.674	.798	.583	.756	.629	.676
	(1,2.5)	.752	.559	.855	.940	.943	.855	.943	.775	.907	.828	.859
	(1,3)	.885	.751	.942	.984	.986	.942	.986	.868	.965	.920	.942
(20,25)	(1,1.5)	.206	.106	.427	.528	.445	.427	.468	.245	.469	.297	.344
	(1,2)	.541	.306	.746	.848	.820	.746	.836	.574	.779	.644	.681
	(1,2.5)	.789	.559	.903	.961	.959	.903	.965	.778	.920	.843	.872
	(1,3)	.914	.760	.967	.991	.993	.967	.994	.870	.974	.930	.952
(25,30)	(1,1.5)	.259	.138	.461	.605	.515	.461	.524	.313	.508	.367	.380
	(1,2)	.649	.433	.783	.900	.889	.783	.395	.700	.817	.754	.736
	(1,2.5)	.887	.724	.932	.981	.985	.932	.985	.879	.945	.927	.914
	(1,3)	.968	.895	.979	.997	.998	.979	.998	.937	.984	.978	.973
(30,30)	(1,1.5)	.289	.187	.420	.631	.558	.420	.554	.383	.511	.406	.395
	(1,2)	.711	.552	.752	.923	.914	.752	.913	.780	.817	.801	.760
	(1,2.5)	.921	.831	.920	.988	.989	.920	.988	.932	.950	.952	.923
	(1,3)	.980	.949	.973	.999	.999	.973	.999	.969	.983	.989	.977

Table.2(c)
Empirical power of tests under Cauchy Distribution

(n_1,n_2)	(σ_1,σ_2)	C	ZH	W
(10,10)	(1,1.5)	.127 .059 .007	.103 .062 .009	.224 .125 .036
	(1,2)	.194 .104 .013	.131 .080 .010	.317 .198 .069
	(1,2.5)	.270 .157 .024	.167 .098 .012	.402 .272 .109
	(1,3)	.343 .212 .041	.207 .119 .013	.476 .340 .152
(10,15)	(1,1.5)	.129 .059 .006	.094 .044 .008	.211 .138 .035
	(1,2)	.202 .106 .013	.114 .049 .008	.321 .225 .075
	(1,2.5)	.289 .166 .026	.147 .059 .008	.408 .308 .123
	(1,3)	.374 .230 .045	.185 .074 .008	.485 .381 .173
(15,15)	(1,1.5)	.156 .081 .014	.124 .061 .012	.201 .126 .039
	(1,2)	.257 .154 .039	.176 .081 .015	.308 .215 .082

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

	(1,2.5)	.377 .248 .074	.241 .123 .019	.401 .296 .135
	(1,3)	.485 .342 .122	.312 .166 .025	.482 .373 .186
(15,20)	(1,1.5)	.150 .081 .014	.110 .050 .006	.249 .137 .040
	(1,2)	.273 .160 .039	.165 .073 .009	.371 .238 .083
	(1,2.5)	.403 .266 .082	.234 .110 .012	.465 .328 .138
	(1,3)	.522 .369 .135	.309 .153 .016	.545 .406 .191
(20,20)	(1,1.5)	.171 .094 .021	.136 .071 .012	.221 .144 .045
	(1,2)	.325 .205 .062	.214 .121 .018	.341 .247 .096
	(1,2.5)	.478 .341 .128	.311 .186 .030	.438 .338 .155
	(1,3)	.612 .467 .207	.410 .264 .050	.515 .417 .224
(20,25)	(1,1.5)	.176 .098 .020	.132 .065 .009	.236 .144 .047
	(1,2)	.341 .217 .064	.215 .111 .014	.353 .248 .104
	(1,2.5)	.497 .361 .140	.314 .176 .022	.456 .342 .175
	(1,3)	.634 .489 .232	.419 .255 .038	.542 .422 .240
(25,30)	(1,1.5)	.203 .118 .031	.148 .069 .014	.229 .140 .046
	(1,2)	.404 .275 .099	.253 .135 .029	.350 .244 .110
	(1,2.5)	.590 .454 .208	.382 .225 .057	.452 .345 .177
	(1,3)	.729 .611 .336	.505 .332 .099	.534 .426 .253
(30,30)	(1,1.5)	.227 .135 .039	.165 .086 .018	.204 .128 .046
	(1,2)	.448 .319 .127	.291 .170 .036	.317 .222 .096
	(1,2.5)	.648 .514 .262	.436 .287 .078	.420 .320 .158
	(1,3)	.782 .675 .408	.572 .413 .135	.509 .403 .225

Table.2(d)

Empirical power of tests under Double Exponential Distribution

(λ_1, λ_2)	(σ_1,σ_2)	C	ZH	V	MZ	W
(10,10)	(1,1.5)	.152 .075 .008	.221 .161 .046	.183 .090 .016	.212 .133 .042	.306 .176 .052
	(1,2)	.276 .153 .023	.409 .326 .123	.328 .178 .036	.363 .237 .085	.497 .327 .122
	(1,2.5)	.407 .245 .047	.571 .486 .216	.455 .266 .062	.496 .345 .134	.651 .473 .204
	(1,3)	.529 .349 .077	.694 .611 .318	.549 .339 .087	.597 .437 .181	.754 .596 .291
(10,15)	(1,1.5)	.161 .074 .007	.239 .152 .053	.152 .060 .006	.216 .130 .046	.294 .200 .050
	(1,2)	.297 .163 .019	.449 .331 .152	.286 .130 .016	.368 .239 .089	.498 .365 .129
	(1,2.5)	.451 .281 .046	.624 .505 .275	.413 .201 .030	.503 .341 .135	.666 .530 .231
	(1,3)	.582 .402 .085	.752 .642 .400	.504 .266 .043	.607 .431 .180	.776 .661 .345
(15,15)	(1,1.5)	.209 .116 .022	.291 .188 .068	.246 .127 .022	.268 .173 .063	.327 .223 .076
	(1,2)	.399 .265 .075	.544 .419 .205	.473 .279 .068	.492 .353 .157	.564 .443 .202
	(1,2.5)	.589 .435 .164	.729 .618 .371	.643 .428 .119	.680 .522 .261	.731 .619 .357
	(1,3)	.726 .588 .271	.845 .759 .526	.749 .534 .177	.794 .657 .356	.836 .748 .500
(15,20)	(1,1.5)	.215 .116 .022	.303 .203 .065	.225 .107 .015	.278 .177 .062	.399 .230 .072
	(1,2)	.437 .291 .079	.579 .438 .213	.451 .246 .045	.512 .356 .162	.654 .461 .204
	(1,2.5)	.639 .483 .179	.776 .662 .389	.624 .388 .088	.696 .525 .264	.811 .652 .369
	(1,3)	.787 .649 .308	.884 .807 .565	.732 .489 .130	.811 .651 .361	.902 .784 .526
	(1,1.5)	.247 .146 .037	.341 .238 .086	.301 .169 .037	.326 .213 .081	.389 .281

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

(20,20)	(1,2) (1,2.5) (1,3)	.517 .367 .137 .727 .595 .291 .858 .757 .468	.651 .540 .285 .837 .760 .506 .927 .877 .691	.591 .394 .116 .766 .577 .210 .859 .697 .297	.610 .465 .219 .796 .665 .379 .898 .793 .508	.098 .654 .544 .275 .827 .743 .475 .913 .855 .647
(20,25)	(1,1.5) (1,2) (1,2.5) (1,3)	.259 .152 .036 .537 .396 .147 .757 .631 .328 .883 .793 .517	.371 .256 .090 .684 .567 .309 .859 .780 .541 .944 .895 .721	.288 .150 .024 .581 .375 .086 .766 .554 .171 .857 .671 .249	.330 .218 .084 .627 .473 .234 .810 .678 .393 .911 .803 .527	.411 .269 .102 .695 .543 .292 .855 .746 .506 .934 .868 .678
(25,30)	(1,1.5) (1,2) (1,2.5) (1,3)	.299 .192 .053 .629 .482 .227 .848 .742 .463 .944 .891 .681	.410 .289 .134 .758 .643 .415 .915 .854 .686 .974 .947 .850	.346 .199 .039 .682 .485 .149 .857 .691 .282 .924 .801 .389	.387 .271 .099 .719 .576 .295 .893 .787 .491 .965 .904 .641	.432 .296 .119 .727 .596 .352 .887 .803 .593 .956 .911 .768
(30,30)	(1,1.5) (1,2) (1,2.5) (1,3)	.326 .211 .064 .684 .548 .276 .883 .797 .554 .961 .924 .760	.436 .318 .142 .804 .705 .460 .942 .903 .745 .983 .966 .892	.409 .250 .065 .747 .583 .240 .907 .782 .415 .959 .875 .541	.429 .292 .126 .766 .630 .377 .922 .839 .602 .977 .934 .759	.427 .310 .132 .733 .619 .382 .890 .821 .624 .955 .921 .794

Figure 1

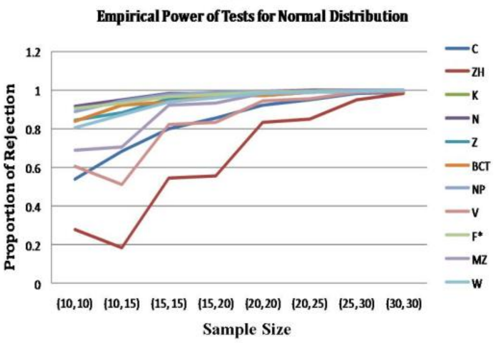


Figure 2

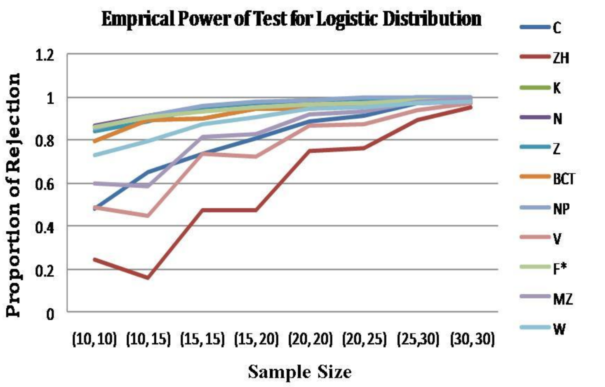


Figure 3

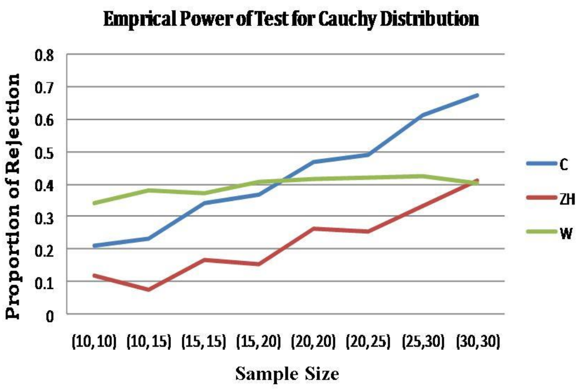
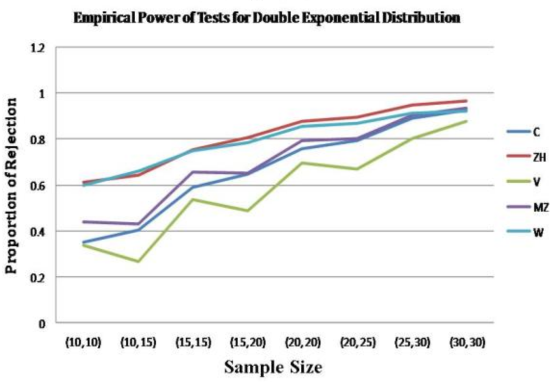


Figure 4



4. DISCUSSION

Under normal distribution, all the eleven considered tests satisfy the specified significance level (10%,5%,1%). In logistic distribution the K, Z, BCT, NP and F^* test are not perfectly able to satisfy the significance level. But in Cauchy distribution, only C test, ZH test and W test able to satisfy the significance level. Under double exponential, among the ten tests only C, ZH, V, MZ and W test able to satisfy the significance level.

Table.2(a) gives the estimate of power of the tests under normal distribution. In this distribution the performance of Newman(N) test is found to be best of the eleven tests. . For the sample size (20,20) both Newman and NP shows the highest power. In case of large sample , the performance of N, Z, NP, F^* and MZ are best under this distribution. ZH appears to be weakest in power under this distribution.

Table.2(b) gives the estimate of power of tests under logistic distribution. It is observed that Newman gives the highest power and ZH gives the less power. In moderate and large samples, Z gives the highest power as N test. We can say that the performance of N, Z and NP tests seemed to be quite well in large sample.

Table.2(c) gives the estimate of power in Cauchy distribution. Under this distribution we calculated the power of only three tests; C test, ZH test and W test. In these three tests, W test gives the highest power and ZH gives the less power. The same performance observed in all sample size. But in sample size (30,30) against the alternative (1,3) C test gives h power.

Table.2(d) gives the estimate of power of tests under double exponential. In this distribution we calculated the power of five tests: C test, ZH test, V test and Z test. Among this four tests ZH test found to be the best in power. But for sample size (10,10), (10,15) W test seemed to be quite well.

Figure1 presents the empirical power of tests for the different sample sizes against the alternative (1,3) under normal distribution. Similarly Figures2 3 and 4 present the empirical power of tests under the logistic, Cauchy and double exponential distributions respectively.

5. CONCLUSION

From the above discussion, we conclude that the performance of Newman (N) test good in both normal and logistic distribution, ZH shows less power under these distribution. Simultaneously the performance of W test is good in Cauchy distribution as well as in double exponential distribution for small sample sizes .But under moderate and large sample sizes the performance of ZH test is well in double exponential but not performed well in Cauchy distribution.

REFERENCES

1. Bliss, C. L., Cochran, W. G. and Tukey, J.W. (1956): A rejection criterion based upon the range. Biometrika, 43, 418-422
2. Brownie, C., Boos, D. D., Hughes-Oliver, J. (1990): Modifying the t and ANOVA F tests when treatment is expected to increase variability relative to controls. Biometrics, 46, 259-266
3. Cucconi, O.(1968) : Un nuovo test non parametrico per it confronto tra due gruppi campionori. Giornale degli Economisti XXVII: 225-248
4. Chen, Y. L. (2004) : A test for two-sample problem based on sample spacings. Tamsui Oxford Journal of Mathematical Sciences, 20(2), 267-278.
5. Lepage,Y.(1971): A combination of Wilcoxon's andAnsari-Bradley's statistics. Biometrika, 58, 213-217.
6. Link,R.F.(1950): The sampling distribution of the ratio of two ranges from independent samples. Ann. Math. Stat., 21(1), 112-116.
7. Leslie, R.T.(1966): Use of range in testing heterogeneity of variance. Biometrika, 53, 221-227.
8. Marozzi, M.(2008): The Lepage location-scale test revisited. Far East Journal of Theoretical Statistics, 24, 137-156.
9. Marozzi, M.(2009): Some notes on the location-scale Cucconi test. Joun. of Nonparametric Stat. 21, 629-647.
10. Marozzi, M.(2013): Nonparametric simulation tests for location and scale testing: A comparison of several methods. Communications in Statistics-Simulation and Computation, 42, 1298-1317.
11. Murakami, H. (2007): Lepage type statistic based on the modified Baumgartner statistic. Computational Statistics and Data Analysis, 51, 5061-5067.
12. Neuhauser, M. (2000): An exact two-sample test based on the Baumgartner-Weiss-Schindler statistic and a modification of Lepage's test. Communication in Statistics-Theory and Methods, 29, 67-78.
13. Newman, D.(1939): The distribution of range in samples from a normal population, expressed in terms of an independent estimate of standard deviation. Biometrika, 31, 20-30.
14. O'Brien, R.G.(1978): Robust techniques for testing heterogeneityof variance effects in factorial design. Psychometrika, 43(3), 327-342.
15. O'Brien, P.C. (1988): Comparing two samples: Extensions of the t, rank sum and long rank tests. J. Amer. Statist. Assoc. 83, 52-61.
16. Overall,J.E. and Woodward,J.A.(1947): A sample test for heterogeneity of variance in complex factorial design. Psychometrika, 39, 311-318.
17. Overall, J.E. and Woodward, J.A.(1976): A robust and powerful test for heterogeneity of variance. University of Texas Medical Branch Psychometric Laboratory.
18. Podgor, M.J. and Gastwirth, J.L. (1994): On non-parametric and generalized tests for the two-sample problem with location and scale change alternatives. Statistics in Medicine, 13, 747-758.
19. Zhang, J. (2006): Powerful two-sample tests based on the likelihood ratio. Technometrics, 48, 95-103

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

Simulated critical values of Some Statistics at 10%,5% and 1% level of significance :

Sample Size	NEWMAN	BLISS-COCHRAN-TUKEY	CADWELL-LESLIE-BROWN
(10,10)	.5526 .6514 .8232	.6484 .6747 .7296	1.8439 2.079 2.6988
(10,15)	.3344 .3752 .4711	.6497 .6762 . 7260	1.8550 2.0880 2.6501
(10,20)	.2390 .2667 .3222	.6566 .6659 .7383	1.9124 2.1838 2.8275
(15,15)	.3673 .4099 .5081	.6266 .6488 .6938	1.6783 1.8470 2.2656
(15,20)	.2608 .2880 .3425	.6244 .6481 .6919	1.6625 1.842 2.2462
(15,25)	.2033 .2227 .2664	.6266 .6501 .6954	1.6777 1.8583 2.2829
(20,20)	.2768 .3058 .3636	.6130 .6328 .6749	1.5842 1.7237 2.0763
(20,25)	.2145 .2337 .2758	.6105 .6501 .6677	1.5673 1.7038 2.0096
(20,30)	.1742 .1900 .2226	.6131 .6343 .6721	1.5847 1.7343 2.0495
(25,25)	.2221 .2432 .2855	.6047 .6260 .6598	1.5295 1.6739 1.9394
(25,30)	.1806 .1955 .2276	.6017 .6197 .6563	1.5106 1.6295 1.9097
(30,30)	.1859 .2016 .2325	.5972 .6161 .6579	1.4826 1.6050 1.8727

Sample Size	LINK	ZHANG ZH	MOWZ
(10,10)	1.6072 1.847 2.3872	2.2516 2.4423 2.4915	3.6705 5.5405 11.1669
(10,15)	1.7924 2.0367 2.6331	2.4282 2.5615 2.6207	3.5728 5.4001 10.7584
(10,20)	1.8974 2.1702 2.8275	2.5471 2.6674 2.7146	3.4659 5.2890 10.4468
(15,15)	1.5006 1.6797 2.0703	2.5361 2.6584 2.7099	3.6685 4.8826 8.9019
(15,20)	1.5867 1.7846 2.2258	2.6153 2.7305 2.7776	3.3216 4.9123 8.7913
(15,25)	1.6496 1.8411 2.2806	2.701 2.7928 2.8323	3.3419 4.9660 8.6945
(20,20)	1.4276 1.5757 1.9297	2.6939 2.7948 2.8303	3.1745 4.6242 8.4162
(20,25)	1.4892 1.641 1.9746	2.7422 2.8375 2.8736	3.1028 4.5311 8.0746
(20,30)	1.5429 1.6984 2.0262	2.7941 2.8755 2.9081	3.1177 4.4983 8.2544
(25,25)	1.3867 1.5283 1.8408	2.7947 2.8758 2.9056	3.0560 4.4794 8.0610
(25,30)	1.4309 1.5673 1.8698	2.8375 2.9037 2.9354	2.9253 4.2825 8.0523
(30,30)	1.3673 1.4891 1.7683	2.8655 2.9321 2.9595	2.8412 4.2963 7.5267

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 4, Issue 11, November 2017

Degrees of Freedom	O'BRIEN
(1,18)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,23)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,28)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,33)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,38)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,43)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,48)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,53)	tv1=2.71 ; tv2=3.84; tv3=6.63
(1,58)	tv1=2.71 ; tv2=3.84; tv3=6.63
(2,27)	tv1=2.3 ; tv2=3!; tv3=4.61
(3,36)	tv1=2.08 ; tv2=2.6; tv3=3.78
(4,45)	tv1=1.94 ; tv2=2.37; tv3=3.32
(5,54)	tv1=1.85 ; tv2=2.21; tv3=3.02
(6,63)	tv1=1.77 ; tv2=2.1; tv3=2.8
(7,72)	tv1=1.72 ; tv2=2.01; tv3=2.64

(m-1),m=Sample Number	OVERALL-WOODWARD Z-VARIANCE
m-1=1	tv1=2.71; tv2=3.84; tv3=6.63
m-1=2	tv1=2.3; tv2=3!; tv3=4.61
m-1=3	tv1=2.08; tv2=2.6; tv3=3.78
m-1=4	tv1=1.94; tv2=2.37; tv3=3.32
m-1=5	tv1=1.85; tv2=2.21; tv3=3.02
m-1=6	tv1=1.77; tv2=2.1; tv3=2.8